A Parametric Study on Concrete Columns Exposed to Biaxial Bending at Elevated Temperatures Using a Probabilistic Analysis

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Abstract. Concrete columns exhibit a loss of both strength and stiffness during fire. The current contribution focusses on the fire performance of concrete columns subjected to biaxial bending in combination with axial loads, as such information is rarely available in literature. First, the second-order effects are quantified using a numerical tool. In Eurocode 2, the most important parameters which influence second-order effects of columns during fire are the slenderness ratio, fire duration as well as the magnitude of the axial loading. The influences of these parameters are investigated using a parametric study. Additionally, from the viewpoint of fire safety and structural reliability, uncertainties should be incorporated in the second-order analysis in order to achieve reliability-based design guidelines. Hence, the numerical tool is further developed in order to take into account uncertainties. Furthermore, the tool is validated using the Eurocode provisions and existing experimental data, while considering an ISO 834 standard fire. Finally, examples are given for the fire resistance design of concrete columns.

Keywords: Concrete columns · Biaxial bending · Fire · Probabilistic analysis

1 Introduction

Fire has an important influence on concrete structures and structural members. Concrete columns, as main structural members, exhibit a loss of both strength and stiffness during fire. Furthermore, columns frequently also have to cope with bending moments from beams and slabs. The current contribution focuses on the fire performance of concrete columns subjected to biaxial bending in combination with axial loads, as such information is rarely available in literature. It is essential to study parameters that influence interaction curves of columns exposed to fire by quantifying uncertainties when taking into account second-order effects.

EN 1992-1-1 (2004) governs the design for concrete structures at ambient temperature and points out that allowance should be made in the design of concrete columns for uncertainties associated with the prediction of second-order effects. Further in EN 1992-1-2 (2004), guidelines for structural fire design are provided. In this

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standard, the design of concrete structures is primarily based on structural member design derived from test results of individual beams, columns and slabs in small furnaces (Bailey 2002). However, a limited number of tests could not cover all the individual cases and full-scale tests are expensive and time-consuming. Therefore, numerical tools have been developed to investigate the behaviour of concrete elements in case of fire. Kodur and Dwaikat (2008) first proposed a numerical model adopting moment-curvature relationships to trace the response of a reinforced concrete beam in case of fire. Furthermore, Van Coile (2015) developed a similar methodology to investigate the mechanical behaviour of concrete slabs and beams during fire exposure in full accordance with EN 1992-1-2 (2004) and taking into account uncertainties. Wang et al. (2016) applied the numerical model to evaluate probabilistic interaction diagrams of columns exposed to fire taking into account only the first-order effects. However, the second-order effects of columns in case of fire still need to be considered. Therefore, the developed numerical tool based on the cross-sectional calculation is presented herein to investigate second-order effects using a probabilistic analysis.

As the first step in this paper, a numerical calculation tool presented in Wang et al. (2016) is developed, taking into account second-order effects. Further, the second-order effects of columns exposed to an ISO 834 standard fire are quantified using this calculation tool while taking into account uncertainties. Since the most important parameters which influence second-order effects of columns during fire are the slenderness ratio, fire duration as well as the magnitude of the axial loading (EN1992-1-2 2004), the influences of these parameters are investigated using a parametric study. Finally, examples are given for the fire design of columns exposed to fire.

2 Basic Calculation Model

A numerical tool was proposed to calculate the combined effect of an axial force (N) and a total bending moment (M) on columns exposed to fire in case of biaxial bending (Wang et al. 2016). This calculation tool is based on a cross-sectional calculation and adopts the stress-strain relationships of concrete and reinforcing bars provided in EN 1992-1-2 (2004). With respect to the cross-sectional calculation, the

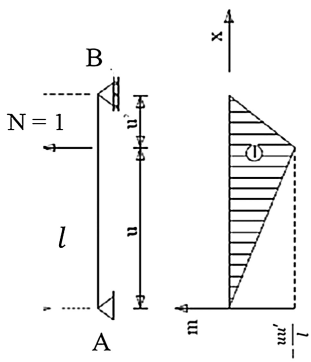

Fig. 1. The bending moment diagram in the case of a vertical force N = 1

basic model has been introduced in Wang et al. (2016). For a given axial load, the moment capacity as well as the moment-curvature relationships can be obtained. As the previous work has been presented, this paper mainly introduces how second-order effects are implemented. A simply supported column is illustrated in Fig. 1, where u is the distance between the calculated point and the support A.

Based on the concepts of virtual work, bending moments m in the case of a vertical force N = 1 can easily be obtained and the deflection v is calculated as:

$$v = \sum \int \mathrm{m}.\frac{\mathrm{M}}{\mathrm{EI}}\mathrm{dx} \tag{1}$$

with M the bending moment at the local cross-section

EI the stiffness of the cross-section

Since the curvature $\chi = \frac{\mathrm{M}}{\mathrm{EI}}$, Eq. (1) can be written as:

$$v = \sum \int \mathrm{m}.\chi\,\mathrm{dx} \tag{2}$$

with χ the curvature

From Eq. (2), it is seen that the $\mathrm{M} - \chi$ relationship is required in order to obtain the deflection along the column. For the first-order analysis, the bending moment is considered to be constant along the column (i.e. corresponding to an axial load with a given eccentricity). However, second-order effects always occur when the column deflects perpendicular to the load axis. An iterative calculation method can be used to perform the second-order analysis.

As the first step of the calculations, the curvature χ in case of the first order bending moment can be obtained based on the cross-sectional calculation. Deflections at any position of the column are calculated according to Eq. (2). Then, additional bending moments caused by deflections under eccentric loads are obtained. Next, a new value for χ corresponding to the new bending moment can be found from a cross-sectional calculation. This procedure is repeated until the bending moment converges and further iterations do not alter the bending moment significantly.

3 A Probabilistic Study Considering Second-Order Effects

In Wang et al. (2016), a simply supported column with the cross-section of 300 mm × 300 mm is considered, with probabilistic models for the basic variables as given in Table 1, in accordance with Holický and Sýkora (2010) and JCSS (2007). As it is explained in Van Coile (2015), a normal distribution may result in physically impossible negative values for the reduction factors of the concrete compressive strength and the reinforcement yield stress at elevated temperatures. Hence, a Beta distribution is proposed which is bounded by three times the standard deviation and the mean value is taken as the nominal value given in EN 1992-1-2 (2004). Here, a Beta distribution with parameters alpha and beta equal to 4 is applied.

Table 1. Probabilistic models for basic variables: property, dimension, distribution, mean value μ, standard deviation σ and coefficient of variation V.

Property	Dim.	Distr.	μ	σ	V
20 °C concrete compressive strength $f_{c,20}$ (f_{ck} = 55 MPa)	MPa	LN	$\frac{f_{ck}}{1-2V_{fc}}$	μ·V	0.15
20 °C reinforcement yield stress $f_{y,20}$ (f_{yk} = 500 MPa)	MPa	LN	$\frac{f_{yk}}{1-2V_{fy}}$	μ·V	0.07
Concrete compressive strength reduction factor $k_{fc}(\theta)$ at temperature θ	–	Beta	EN 1992-1-2 (2004)	μ·V	Van Coile (2015)
Reinforcement yield stress reduction factor $k_{fy}(\theta)$ at temperature θ	–	Beta	EN 1992-1-2 (2004)	μ·V	Van Coile (2015)
Concrete cover c	mm	Beta	25	5	σ/μ
Column width z	mm	DET	z_{nom}	300	–

Details of calculations have already been introduced in Wang et al. (2016) as well as the cross-sectional moment capacity in case of fire. However, the second-order effects of columns cannot be neglected. Therefore, the numerical tool is adopted herein and the most important parameters which influence second-order effects of columns during fire (i.e. the ISO 834 standard fire)—the slenderness ratio, fire duration as well as the magnitude of the axial loading (EN 1992-1-2 2004)—are investigated.

As a first step of the study, the cross-sectional bending moment capacity of columns (reinforcement ratio of 0.5) for different axial loads which are located in the diagonal axis of the cross-section are considered in case of an ISO 834 standard fire of fire durations of 30 min and 90 min are shown in Fig. 2 and Fig. 3, respectively, based on 10000 Monte Carlo simulations of the cross-sectional calculation (Wang et al. 2016). The dashed lines are the maximum and the minimum observed values.

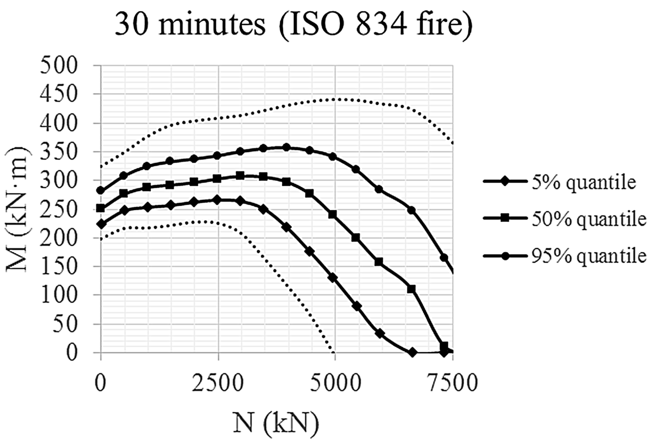

Fig. 2. Interaction curves of the cross-section exposed to an ISO 834 fire of 30 min including uncertainties

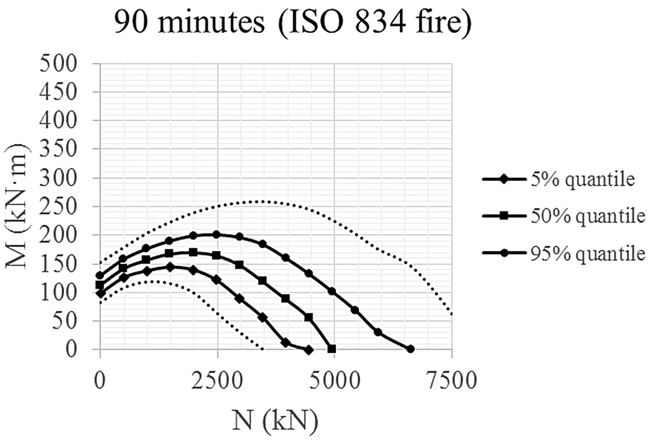

Fig. 3. Interaction curves of the cross-section exposed to an ISO 834 fire of 90 min including uncertainties

From Figs. 2 and 3, it is observed that the bending moment capacity decreases in function of the fire duration. Considering uncertainties of the variables presented in Table 1, the range of the moment capacity changes very significantly in case of different axial loads. Based on the results shown in Figs. 2 and 3, it is observed that the range of the moment capacity in case of a small axial load is rather small compared with that in case of a large axial load. The standard deviation of the moment capacity increases significantly beyond an axial load of 2500 kN in case of 30 min fire exposure and 1000 kN in case of 90 min fire exposure. This is related with the observation that when the applied load reaches this value, the minimum observed value begins to decrease with increasing axial load while the maximum observed value still increases. The load corresponding to the maximum moment capacity of the minimum observed value is 2500 kN in case of 30 min fire exposure and 1000 kN in case of 90 min fire exposure while the maximum observed value is 5000 kN in case of 30 min fire exposure and 3500 kN in case of 90 min fire exposure.

Further, the second-order effects of columns are taken into account, based on the cross-sectional calculation as explained in Sect. 2. The interaction diagrams associated to different lengths of columns (3 m and 4 m) exposed to fire durations of 30 min are presented in Figs. 4 and 5, respectively.

In Figs. 4 and 5, it is observed that the bending moment capacity decreases with increasing axial load. Unlike interaction diagrams of the cross-section shown in Figs. 2 and 3, the range of the moment capacity varies slightly with increasing axial load. It is worth mentioning that in Fig. 2, the range of the moment capacity increases significantly beyond an axial load of 2500 kN. However, this phenomenon is not observed for both of the columns in Figs. 4 and 5. Further, the 4 m long column fails around 3500 kN while the 3 m long column could bear more than 4000 kN axial load. From the indicated comparison with Figs. 2 and 3, it is seen that it is important to consider second-order effects of columns exposed to fire.

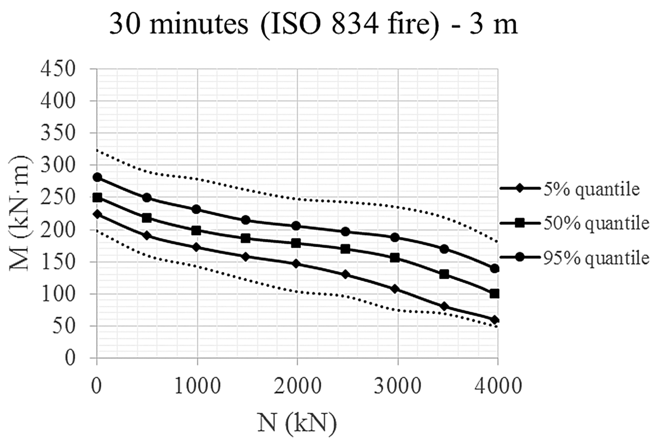

Fig. 4. Interaction curves of a column (3 m long) exposed to an ISO 834 fire of 30 min including uncertainties and second-order effects

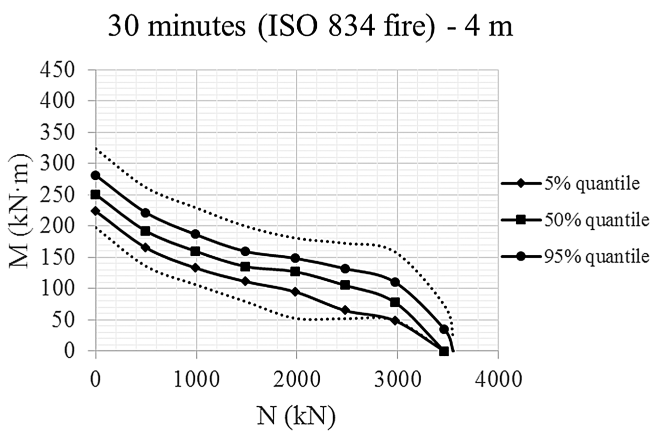

Fig. 5. Interaction curves of a column (4 m long) exposed to an ISO 834 fire of 30 min including uncertainties and second-order effects

The similar interaction curves of these columns in case of a 90 min standard fire exposure for a column length of 3 m, and 4 m are shown in Fig. 6 and Fig. 7, respectively.

The same phenomenon is observed as in case of 30 min fire exposure, i.e. the bending moment capacity keeps decreasing with increasing axial load for both of the columns in case of a fire duration of 90 min. For both of the columns, the moment capacity firstly drops significantly beyond an axial load of 500 kN. Then, the decrease reduces between an axial load of 500 kN and 1000 kN. Finally, the moment capacity decreases significantly until the columns fail. This decrease on the moment capacity is more significant than that in case of a 30 min fire exposure.

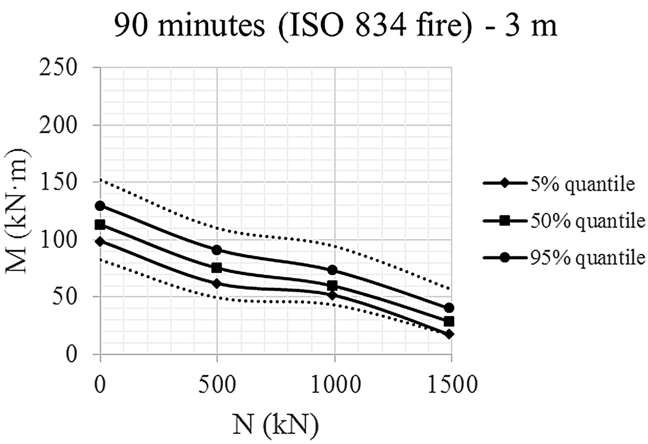

Fig. 6. Interaction curves of a column (3 m long) exposed to an ISO 834 fire of 90 min including uncertainties and second-order effects

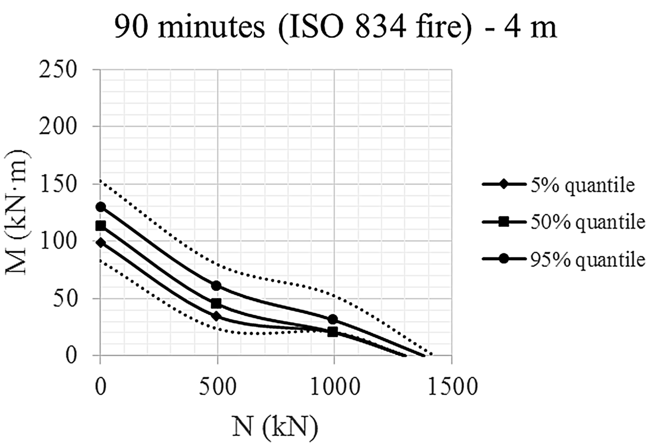

Fig. 7. Interaction curves of a column (4 m long) exposed to an ISO 834 fire of 90 min including uncertainties and second-order effects

Finally, considering different axial loads, the 5%-quantile, 50%-quantile and 95%-quantile maximum permitted eccentricities for the investigated columns of 3 m and 4 m lengths are determined in case of fire durations of 30 min and 90 min. The results are provided in Table 2 and Table 3, respectively.

Table 2. The 5%-quantile, 50%-quantile and 95%-quantile maximum permitted eccentricities for the investigated columns in case of a fire duration of 30 min

Case	Maximum permitted eccentricity (mm)														
	N = 495 kN			N = 990 kN			N = 1485 kN			N = 1980 kN			N = 2475 kN		
Length (m)	Quantile														
	5%	50%	95%	5%	50%	95%	5%	50%	95%	5%	50%	95%	5%	50%	95%
3	38.22	43.91	50.13	17.11	19.82	23.07	10.35	12.24	14.13	7.10	8.72	10.07	4.93	6.56	7.64
4	33.08	38.49	44.45	13.05	15.76	18.46	7.10	8.72	10.35	4.39	6.02	7.10	2.23	3.85	4.93

Table 3. The 5%-quantile, 50%-quantile and 95%-quantile maximum permitted eccentricities for the investigated columns in case of a fire duration of 90 min

Case	Maximum permitted eccentricity (mm)								
	N = 495 kN			N = 990 kN			N = 1485 kN		
Length (m)	Quantile								
	5%	50%	95%	5%	50%	95%	5%	50%	95%
3	12.24	14.95	18.19	4.93	5.74	7.10	1.19	1.96	2.72
4	6.56	8.72	11.97	1.68	1.68	2.77	–	–	–

4 Conclusion

Based on the investigation of second-order effects, it is concluded that the second-order effects are more pronounced with increasing fire duration and with increasing slenderness ratio. Taking into account the uncertainties associated with the column characteristics, the results show that the range of the cross-sectional moment capacity changes significantly for different axial loads while the standard deviation of the moment capacity of columns varies less significantly when second-order effects are considered.

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